

# Optimising the Unpredictability of a Ballistic Missile

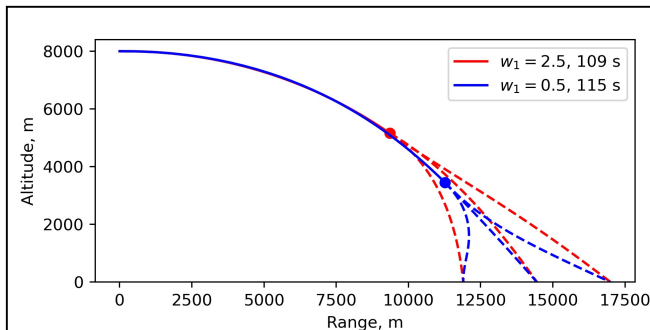
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**Abstract:** Modern missile threats employ a variety of techniques to defeat countermeasures. Hence, to understand missile capabilities, it is necessary to comprehend the impact of evasion behaviours on missile performance. The general forms of such behaviours are presented in literature [1], but it is without revealing their detriment on performance. This motivated relating evasion behaviours to mathematical objectives that promote evasion and revealing the interdependencies between evasion and performance. This capability can be used to predict optimal behaviours given relative importance of performance measures. Preliminary investigation is concerned with guidance that attempts to minimise the ability of an observer to predict the target location. More specifically, if many possible range targets share an initially common route, the true target may be disguised.

We modelled a ballistic glide projectile in two dimensions on a flat Earth using arbitrary parameters and three degrees of freedom. The projectile is subject to aerodynamic drag and lift forces controllable via angle of attack, and to constraints applied on the maximum lift coefficient and angular rates. The Pyomo Differential Algebraic Equations (DAE) [2] framework was used to represent the model as a constrained nonlinear optimisation problem, where the dynamics are discretised by finite differences with boundary conditions applied to both initial and final states. Modelling the projectile involved formulating a set of two-stage flight paths, where each trajectory shares the initial path before branching off to reach a different target range. This allows us to minimise the total time taken to reach the target in addition to the times for which the vehicle strays from the common path. Weighting these times facilitates adjustment of the relative value between having a long-shared initial stage and the capability to target vaster ranges.



**Figure 1: Min, mid and max range targets obscured by an initially common trajectory. The coefficient  $w_1 = 2.5$  aims for a shorter time to reach targets, whereas  $w_1 = 0.5$  seeks to reduce target predictability.**

With a functioning numerical solver, the system was used to investigate the effect that maximum and minimum ranges have on the common initial stage duration, and to examine the trade-off between the duration of the common stage and the overall flight time. A sample set of trajectories produced by the solver is presented in Figure 1, depicting the consequence of faster routes on path predictability.

The framework has produced useful insight into evasion behaviour, and Pyomo DAE is sufficiently expressive to implement the governing system equations simply. Extensions to

implement higher fidelity dynamics or additional flightpath constraints are straightforward, and further investigation into other evasion manoeuvres is underway.

- [1] Li, G., Zhang, H., Tang, G. (2015). Maneuver characteristics analysis for hypersonic glide vehicles. *Aerospace Science and Technology*, 43, 321-328. <https://doi.org/10.1016/j.ast.2015.03.016>
- [2] Sandia National Laboratories (2017). *Pyomo Documentation: Dynamic Optimization with pyomo.DAE*. [https://pyomo.readthedocs.io/en/stable/modeling\\_extensions/dae.html](https://pyomo.readthedocs.io/en/stable/modeling_extensions/dae.html)

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