# A variable-rate sampling framework for state-space coverage in CFD applications

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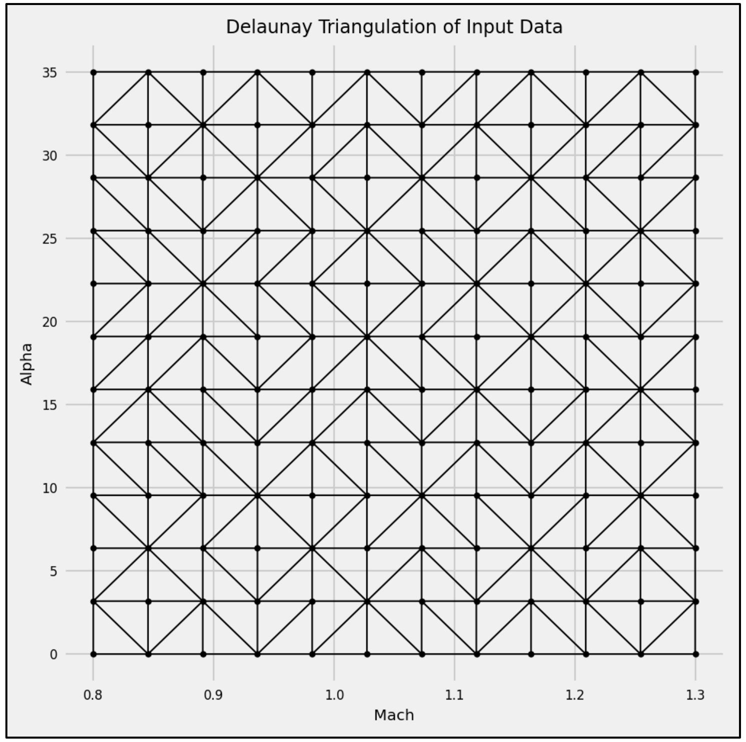
# Computational Fluid Dynamics & Sampling

- When designing an aircraft, you are interested in performance capabilities – e.g. lift coefficient
- Want to **explore the whole solution space** (Mach, AoA) to understand the aircraft performance in all possible flight conditions
- Want to **sample the least number of points** possible – CFD is computationally expensive

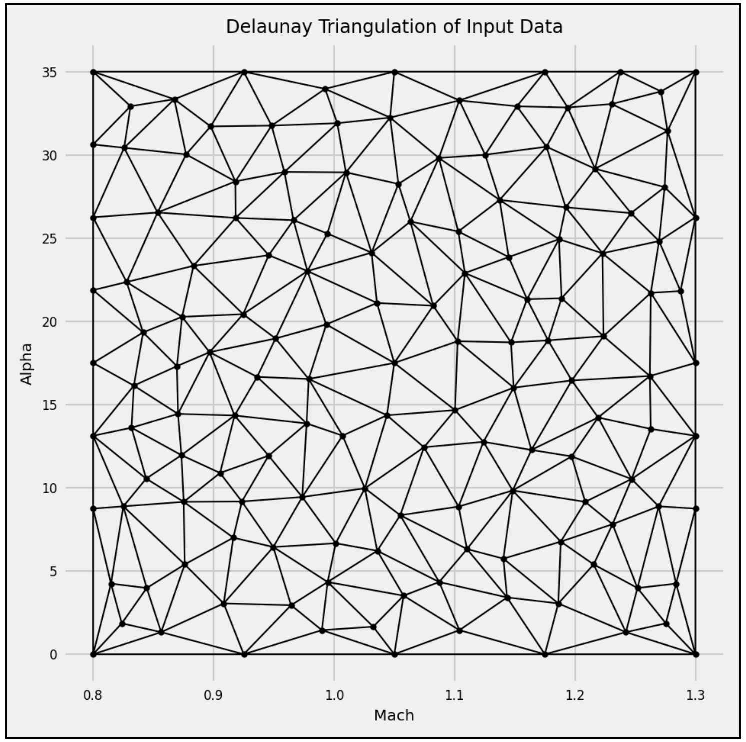
Achieving sufficient exploration with the minimum number of samples is a balancing act!

# Sampling Methods

Grid Sampling




Volume-filling Mesh Sampling (VFM)




Error-based Mesh Sampling (EBM)

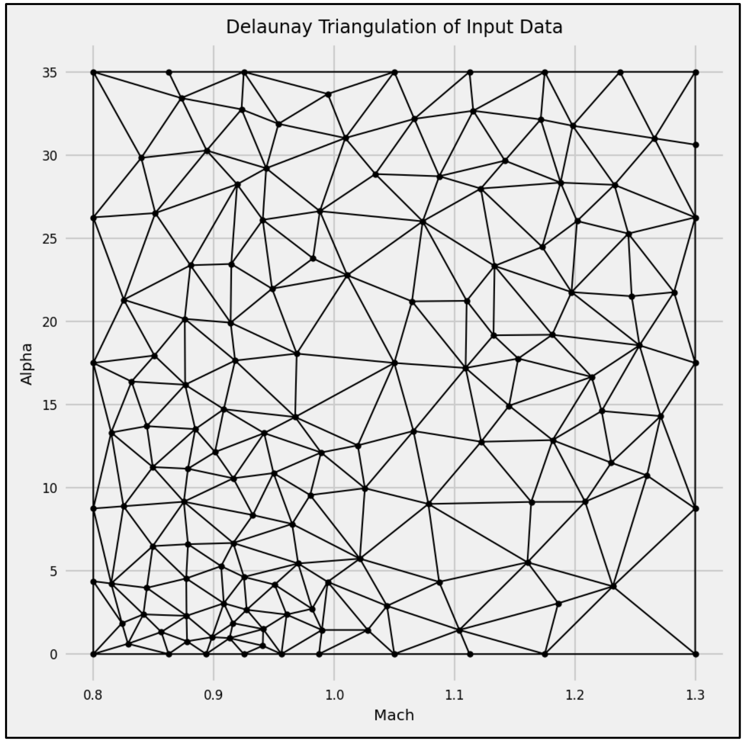



**Figure 1**: Examples of a grid sampler (left), volume-filling mesh sampler (middle) and error-based mesh sampler (right)

# Error-Based Mesh Sampler

- Error calculations:

$$U = \frac{\Sigma\varepsilon}{n_{pts}} L$$

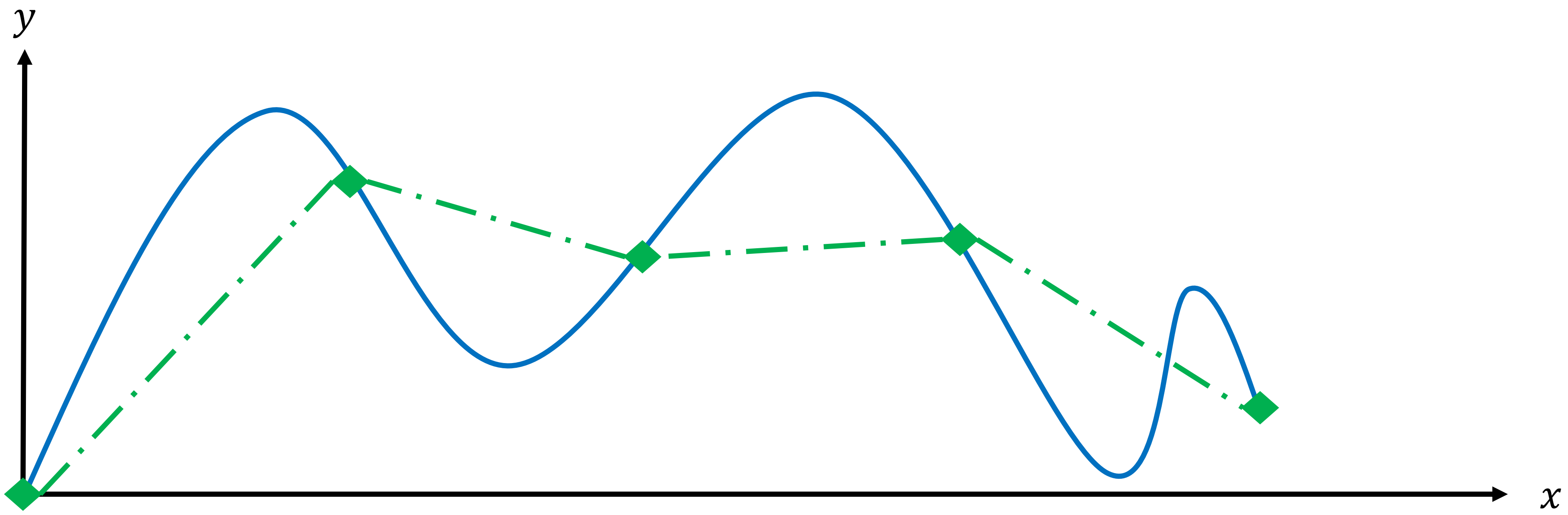
Where:

- $U$ is the utility for ranking samples
- $\varepsilon$ is the error at a single point
- $n_{pts}$ is the number of points surrounding a line/simplex that aren't part of the convex hull
- $L$ is the edge length of the cell (or volume for simplices)

$x$

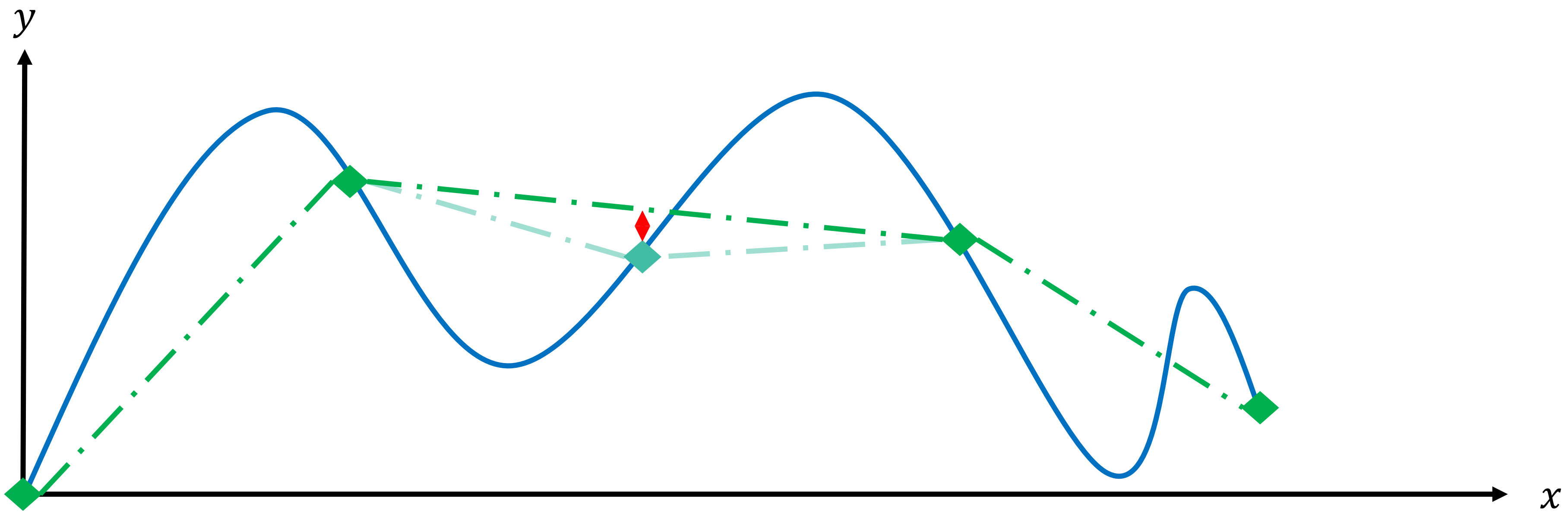
# Error-Based Mesh Sampler

- Before each new point is added, the sampler evaluates the error of a point by evaluating the impact on the interpolation at that point should it be removed.



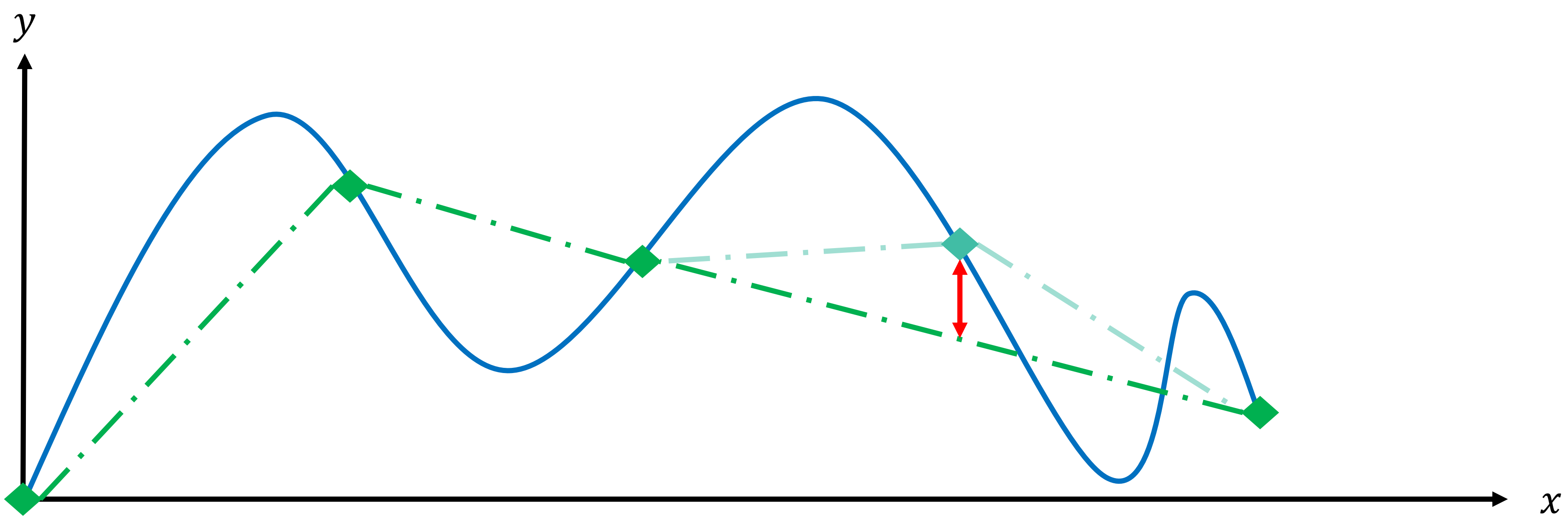
# Error-Based Mesh Sampler

- If we check the middle point, we see that taking it out makes little impact on the interpolation at that value of x



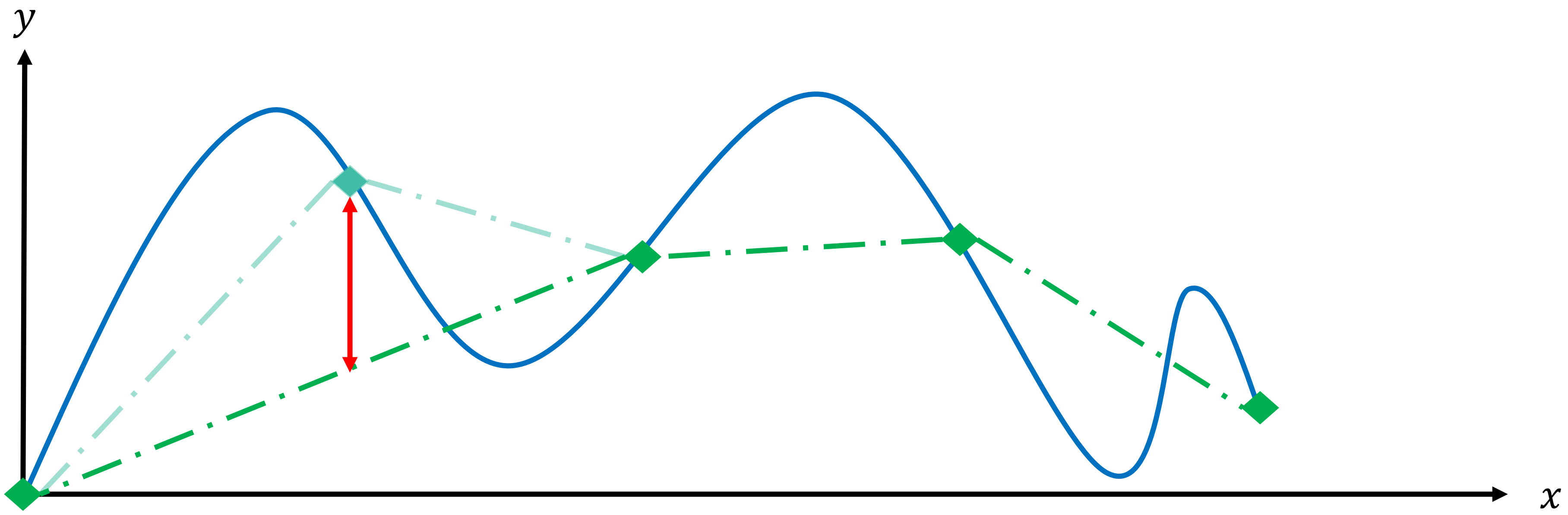
# Error-Based Mesh Sampler

- Looking at the right point, it being removed has a much bigger impact



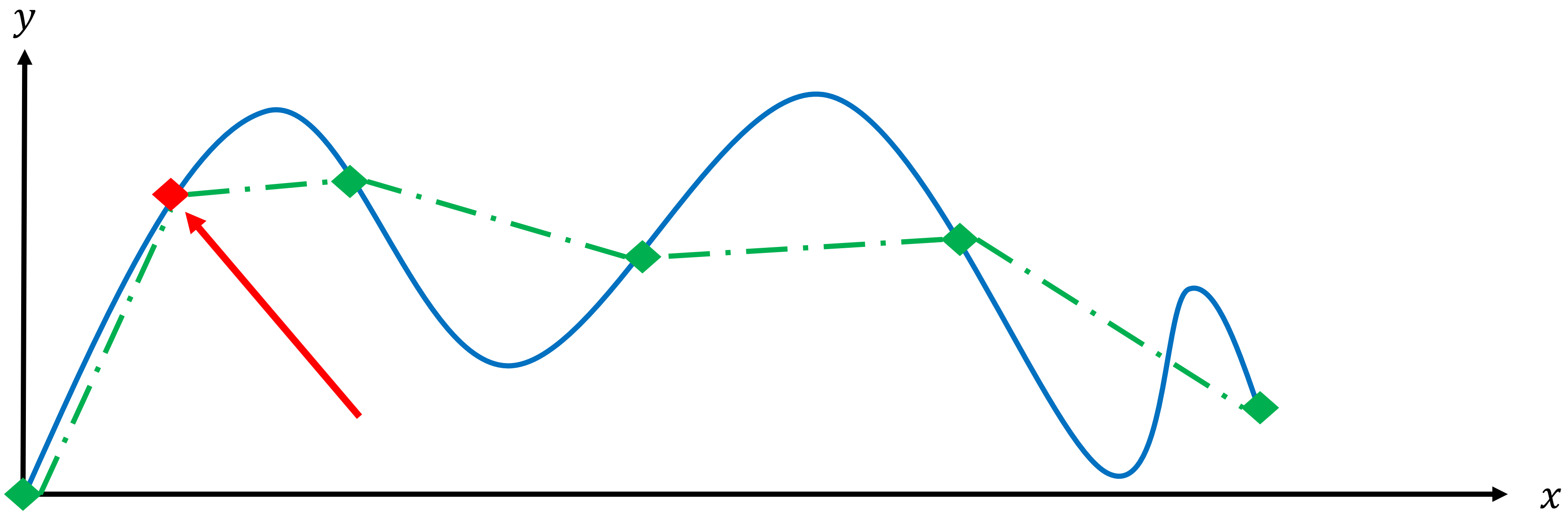
# Error-Based Mesh Sampler

- But the left-hand point has the greatest negative impact on the interpolation performance, so we want to sample around there.



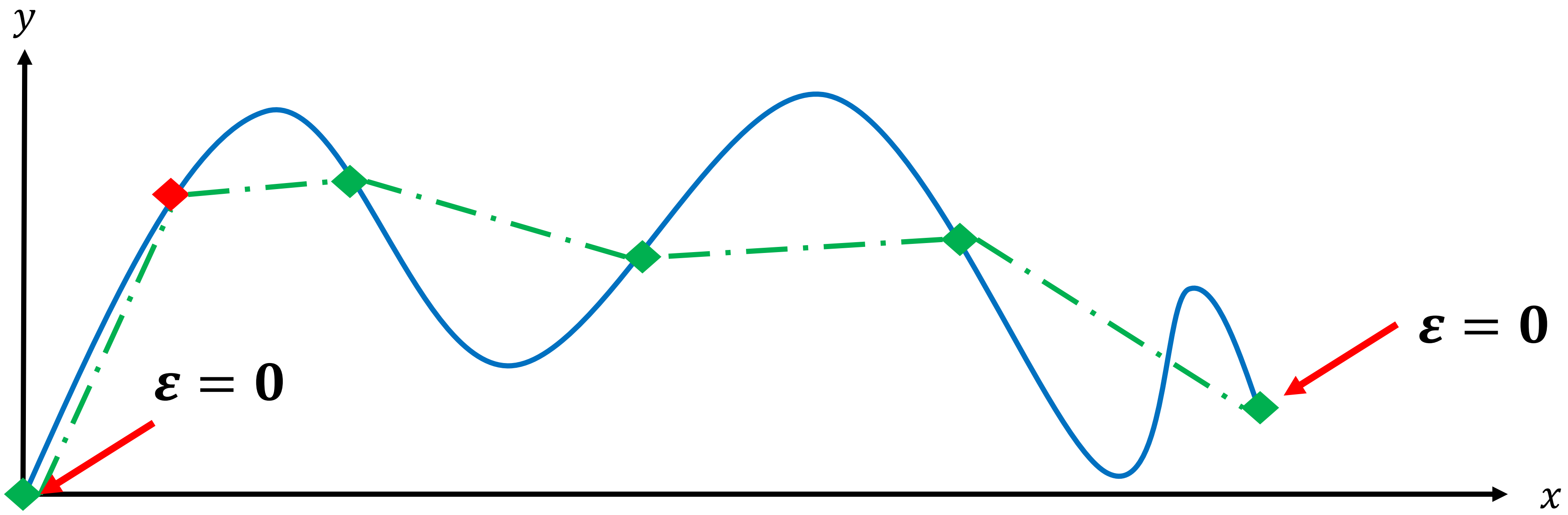
# Error-Based Mesh Sampler

- So, the sample will look to sample around this point. As there are two options, it will look at the one that has the greatest distance between points (or volume in a simplex)

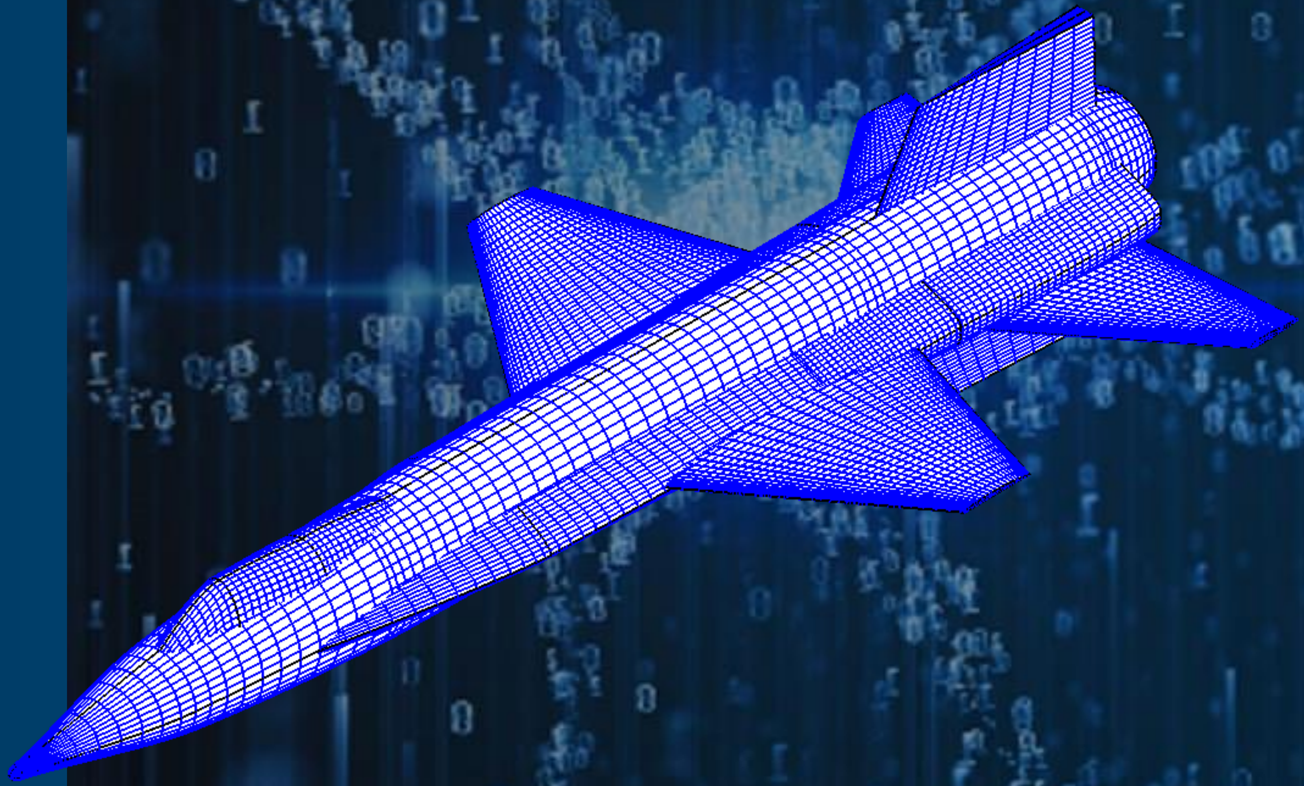


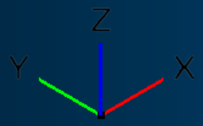
# Error-Based Mesh Sampler

$$U = \frac{\Sigma\varepsilon}{n_{pts}} L$$

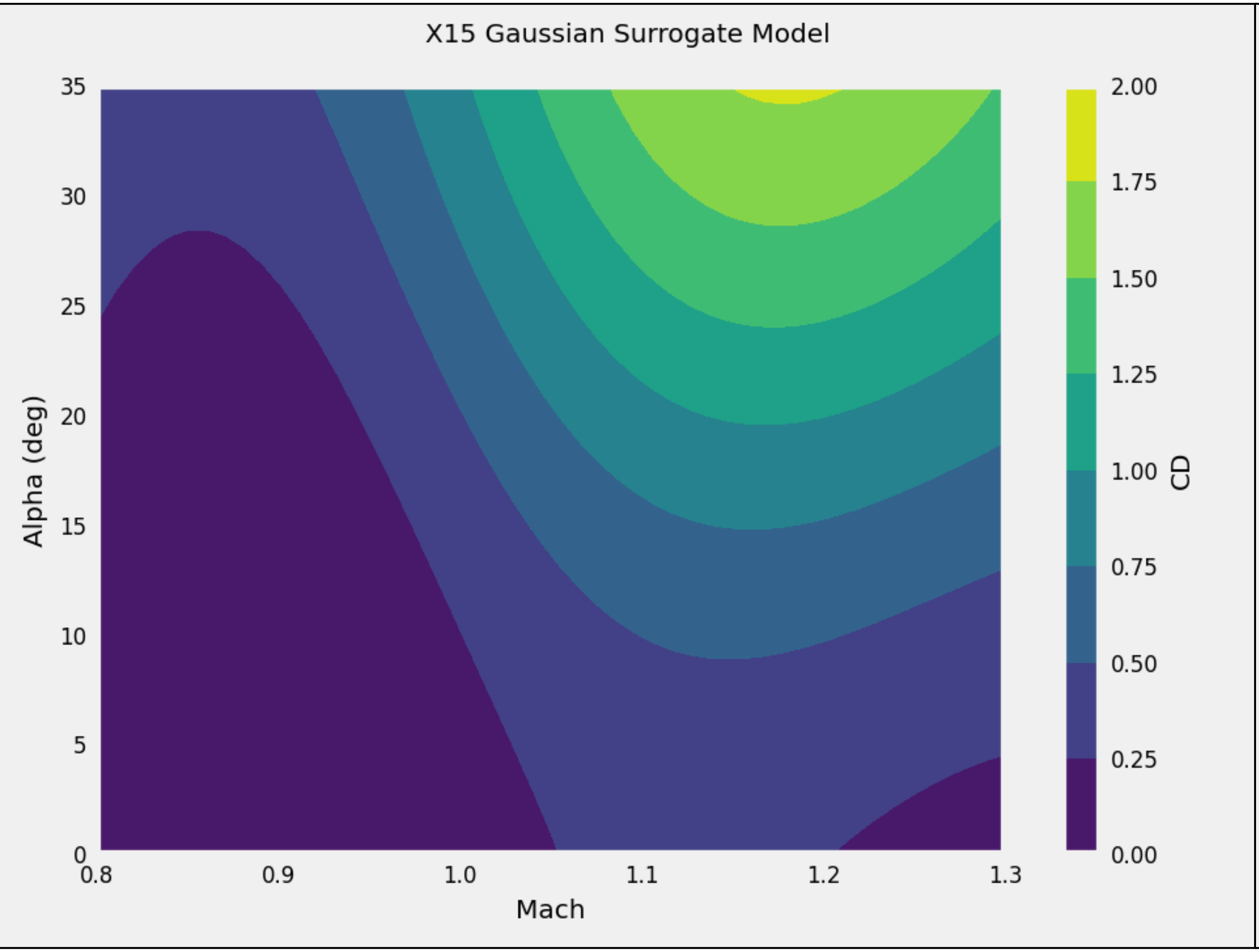


# X15 Test Case





# X15 Drag (CD) and Lift (CL) Coefficients



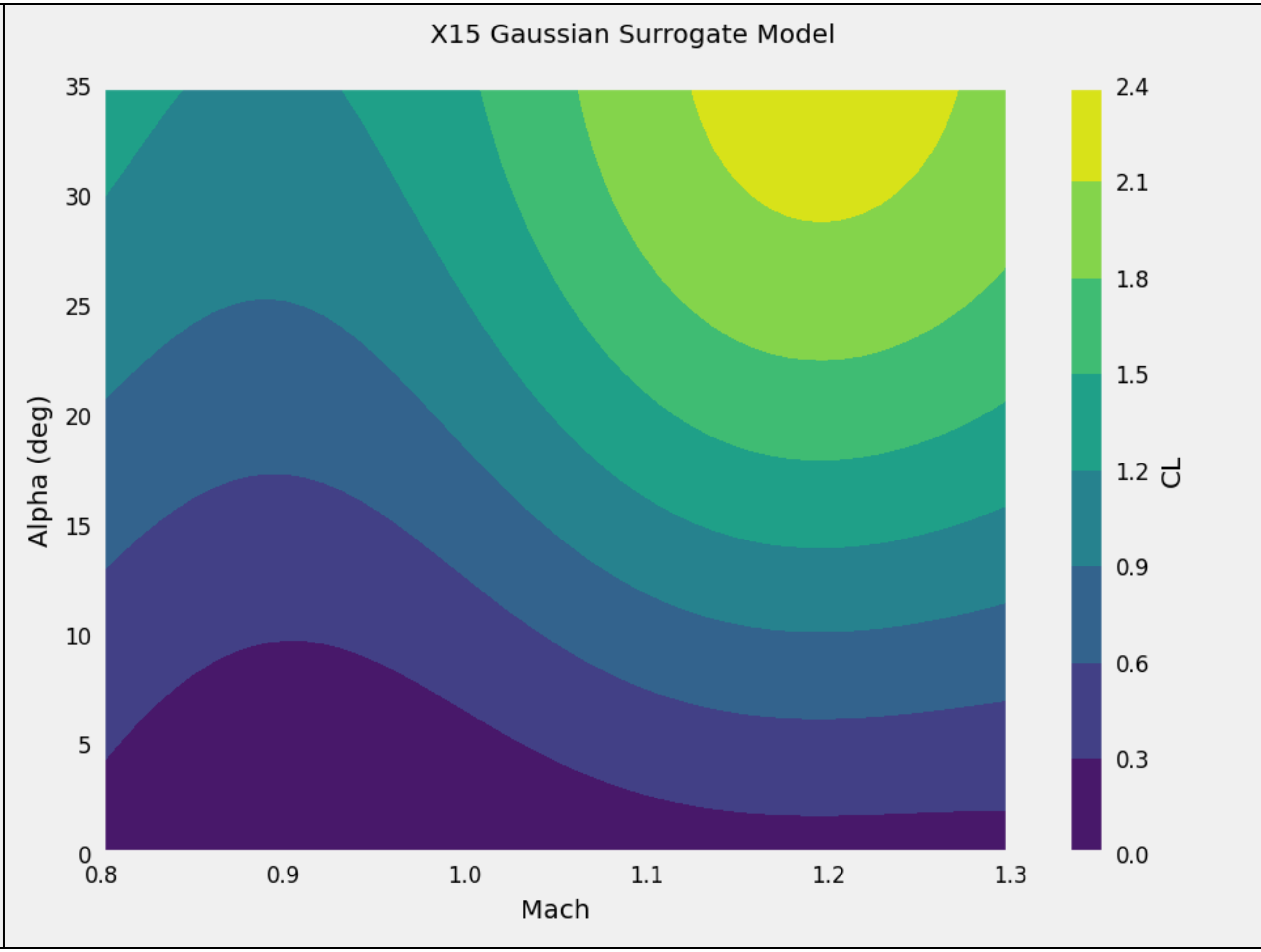
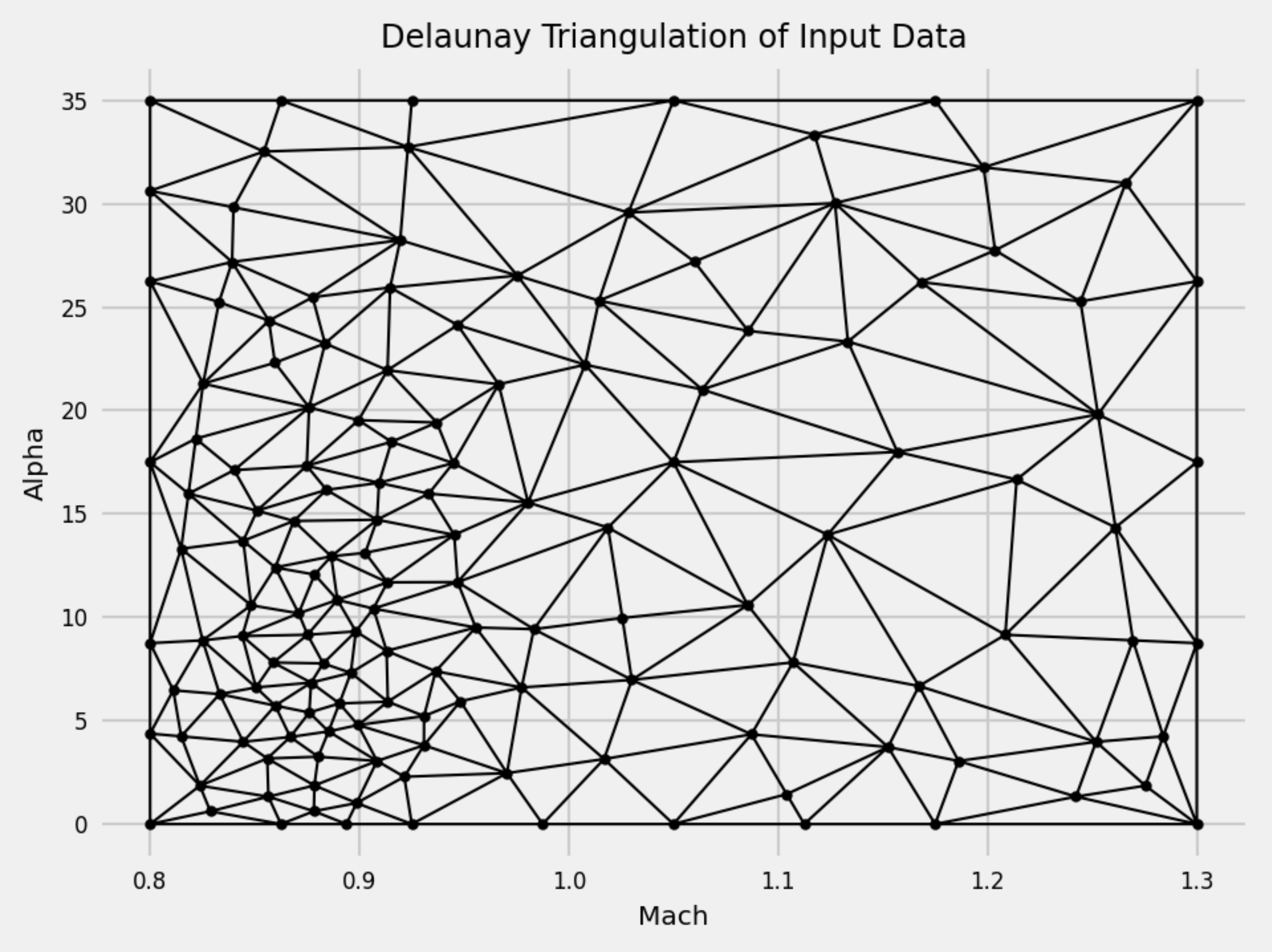




**Figure 2:** Drag coefficient of an X15 for different values of angle of attack and Mach number (left), and a Gaussian surrogate of the drag coefficient (right)

# Drag Coefficient – Visualisation



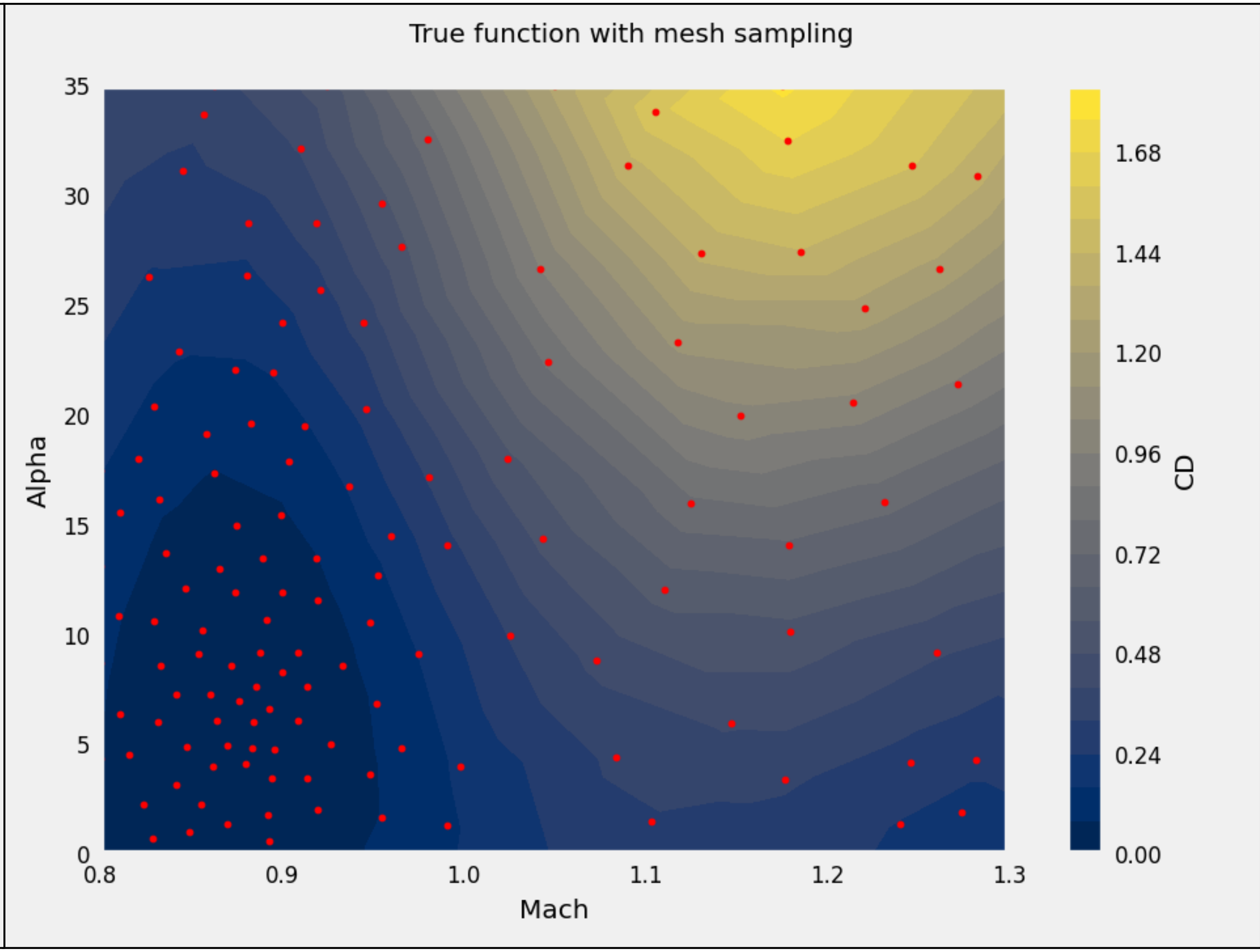




**Figure 3:** Error-based sampler mesh (left) and error-based mesh sampler points and interpolated function (right)

# Results – Drag Coefficient

| Error-Based Mesh | | |
|---|---|---|
| Average $\varepsilon$ | % Fewer Samples vs Grid | % Fewer Samples vs VFM |
| 50% | 44% | 0% |
| 20% | 36% | 46% |
| 10% | 49% | 42% |

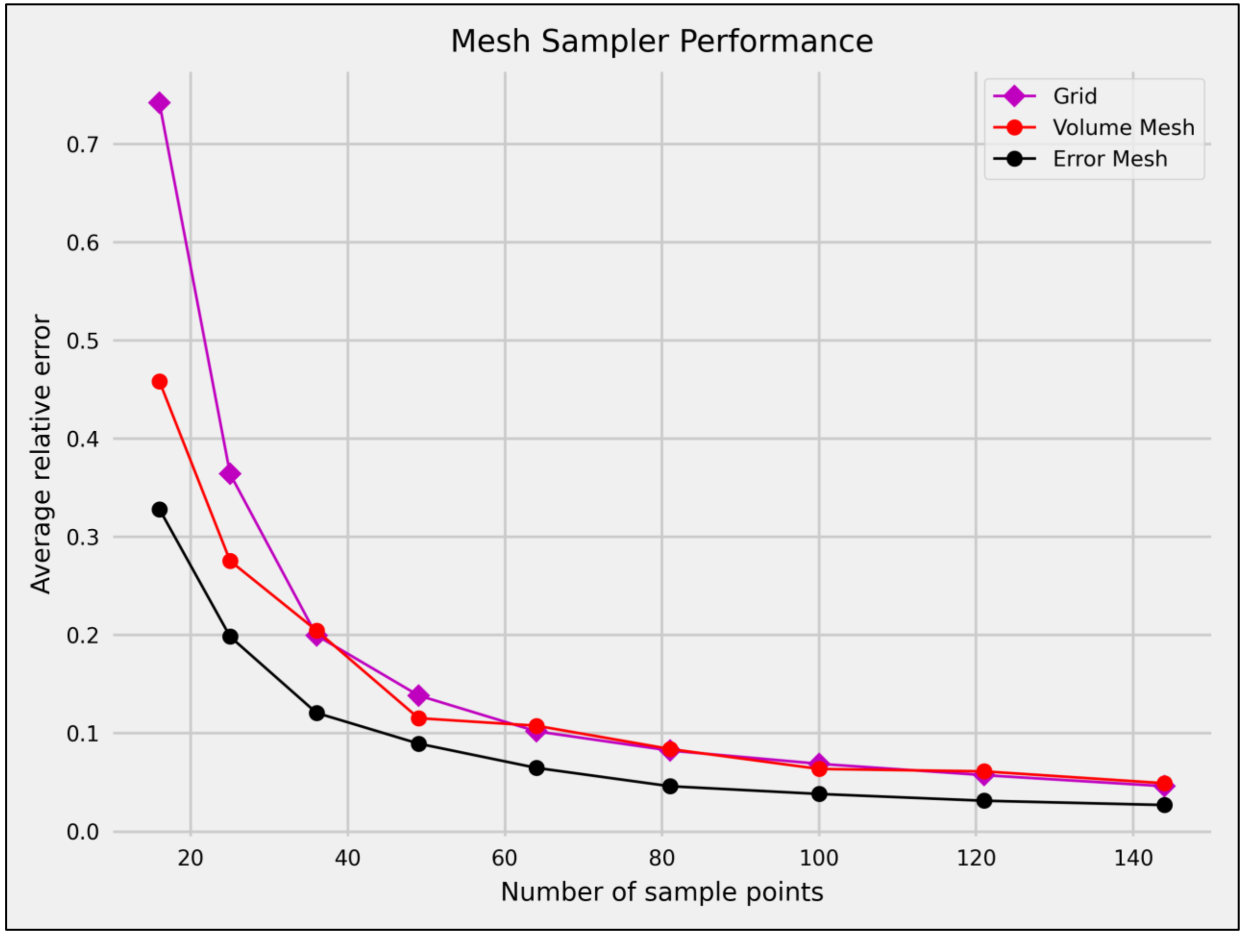



**Figure 4:** Performance of the error-based mesh sampler against the volume-filling mesh and the grid sampler for CD

# Lift Coefficient – Visualisation

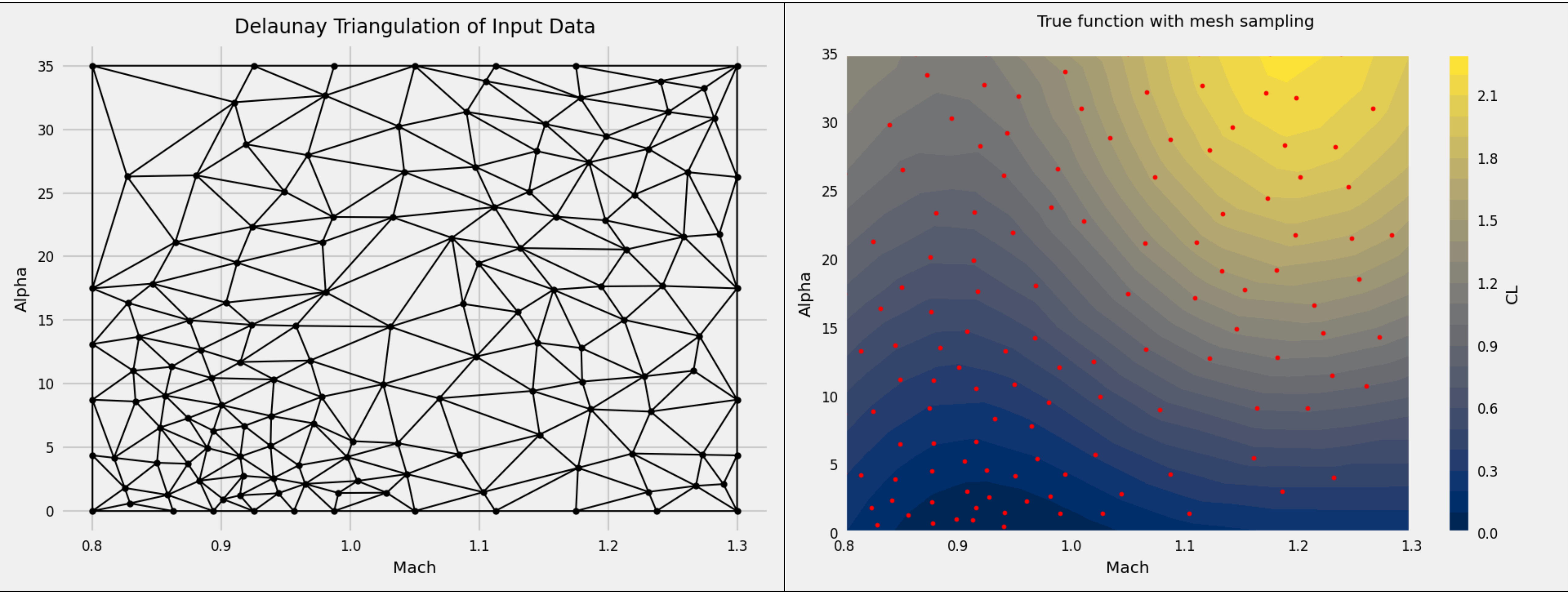



**Figure 5:** Error-based sampler mesh (left) and error-based mesh sampler points and interpolated function (right)

# Results – Lift Coefficient

| Error-Based Mesh | | |
|---|---|---|
| Average $\varepsilon$ | % Fewer Samples vs Grid | % Fewer Samples vs VFM |
| 15% | 0% | 77% |
| 10% | 0% | 65% |
| 5% | 0% | 0% |

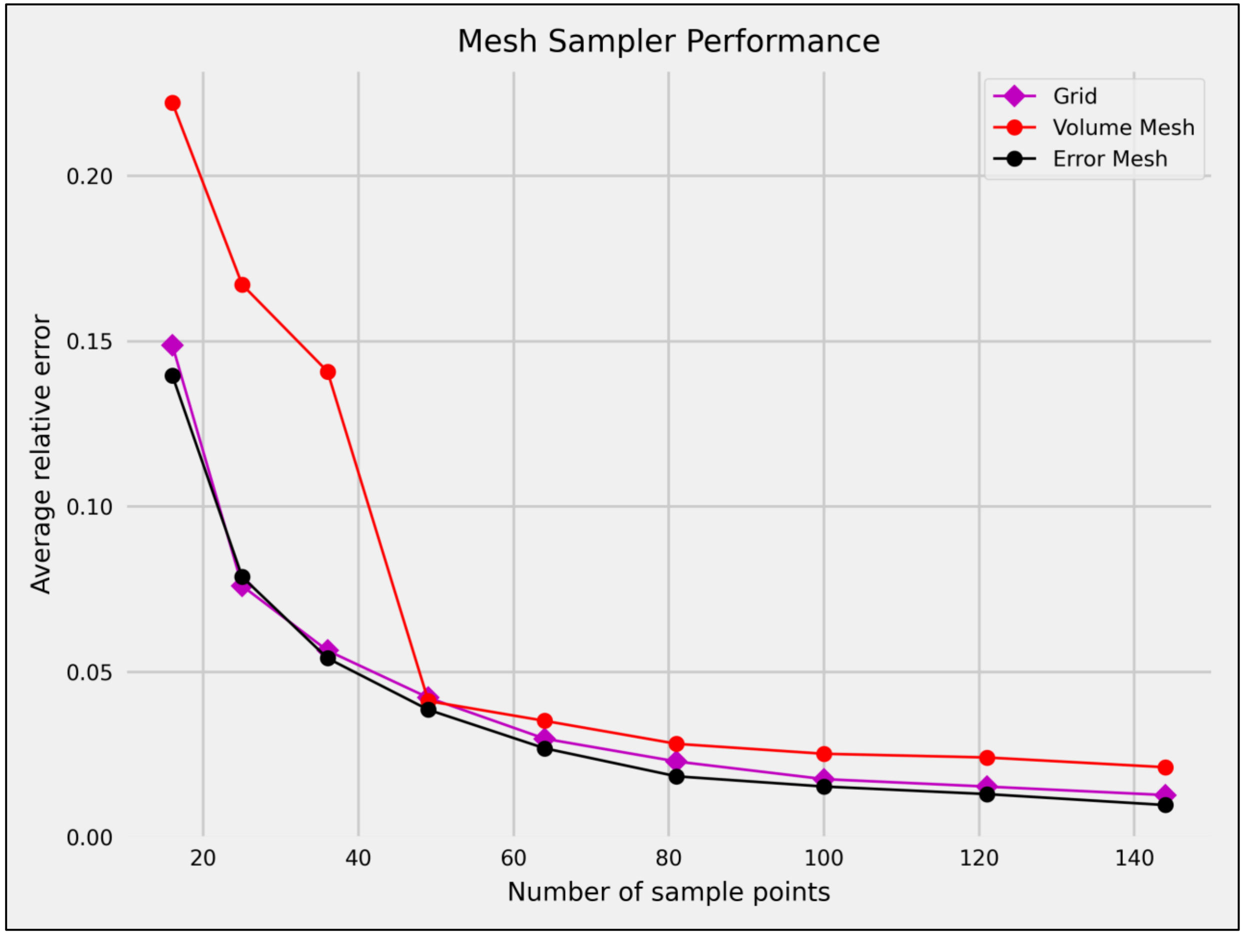



**Figure 6:** Performance of the error-based mesh sampler against the volume-filling mesh and the grid sampler for CL

# Results – Lift Coefficient

- A more refined number of samples shows improvements with the error-based mesh

| Error-Based Mesh | |
|---|---|
| Average $\varepsilon$ | % Fewer Samples vs Grid |
| 15% | 0% |
| 10% | 22% |
| 5% | 20% |

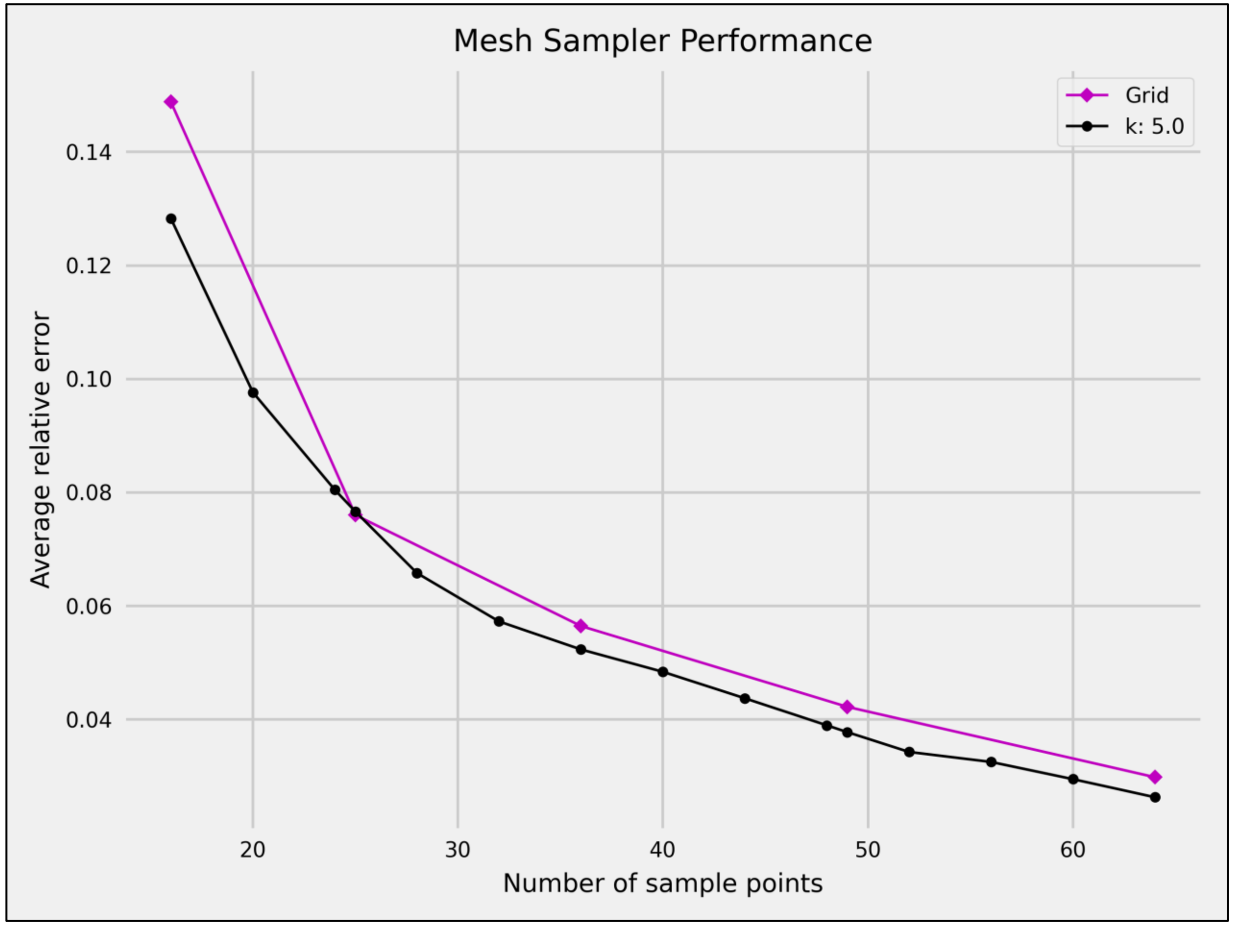



**Figure 7:** Performance of the error-based mesh sampler (k=5.0) against the grid sampler for CL

# Results – Lift Coefficient

| Error-Based Mesh | | |
|---|---|---|
| Average $\varepsilon$ | % Fewer Samples vs Grid | % Fewer Samples vs VFM |
| 15% | 0% | 77% |
| 10% | 0% | 65% |
| 5% | 0% | 0% |

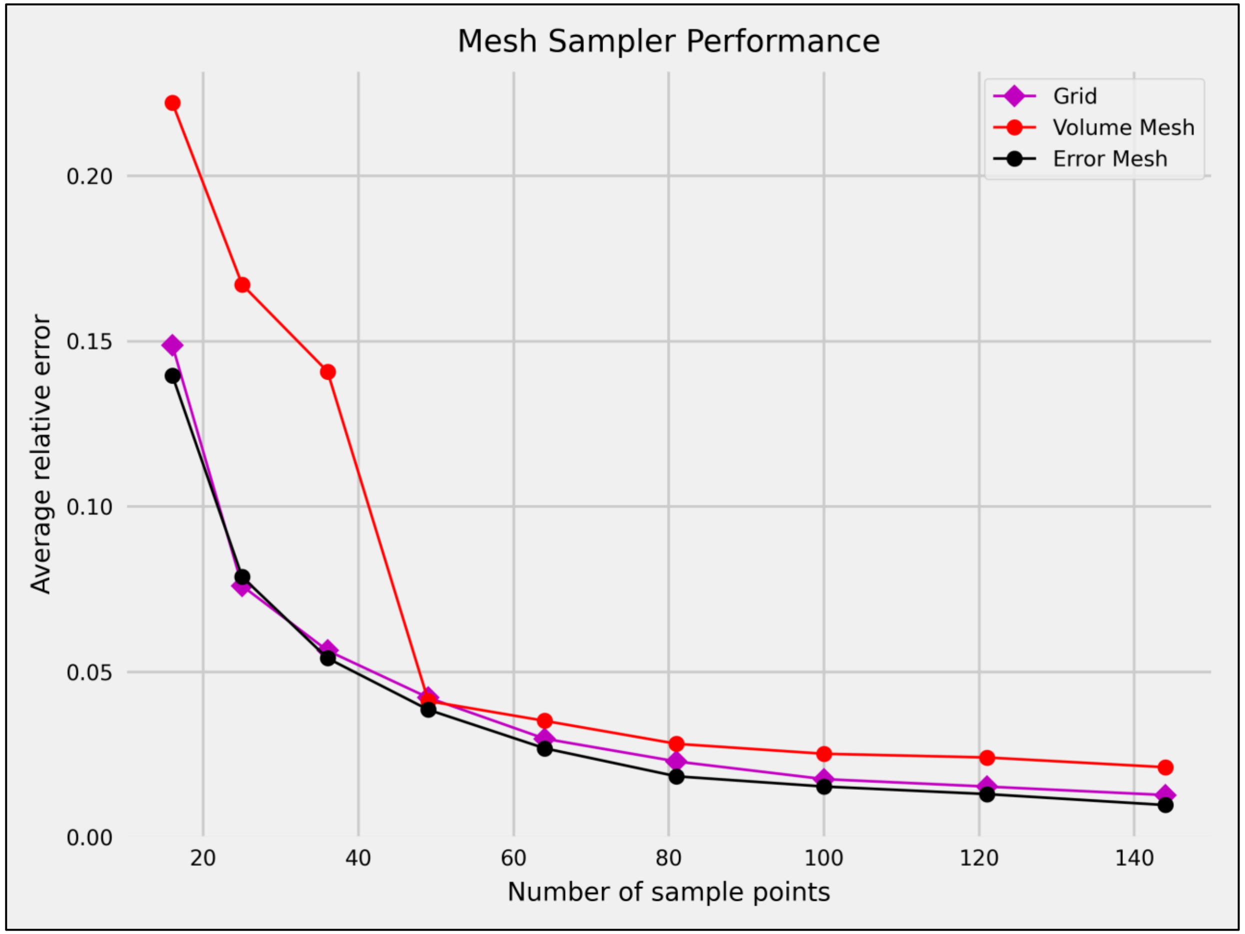



**Figure 8:** Performance of the error-based mesh sampler against the volume-filling mesh and the grid sampler for CL

# Results – Lift Coefficient

- Non-linearity at the boundary is harder for the mesh-samplers as it is part of convex hull

- This is supported by looking at the error distribution for our volume-filling mesh

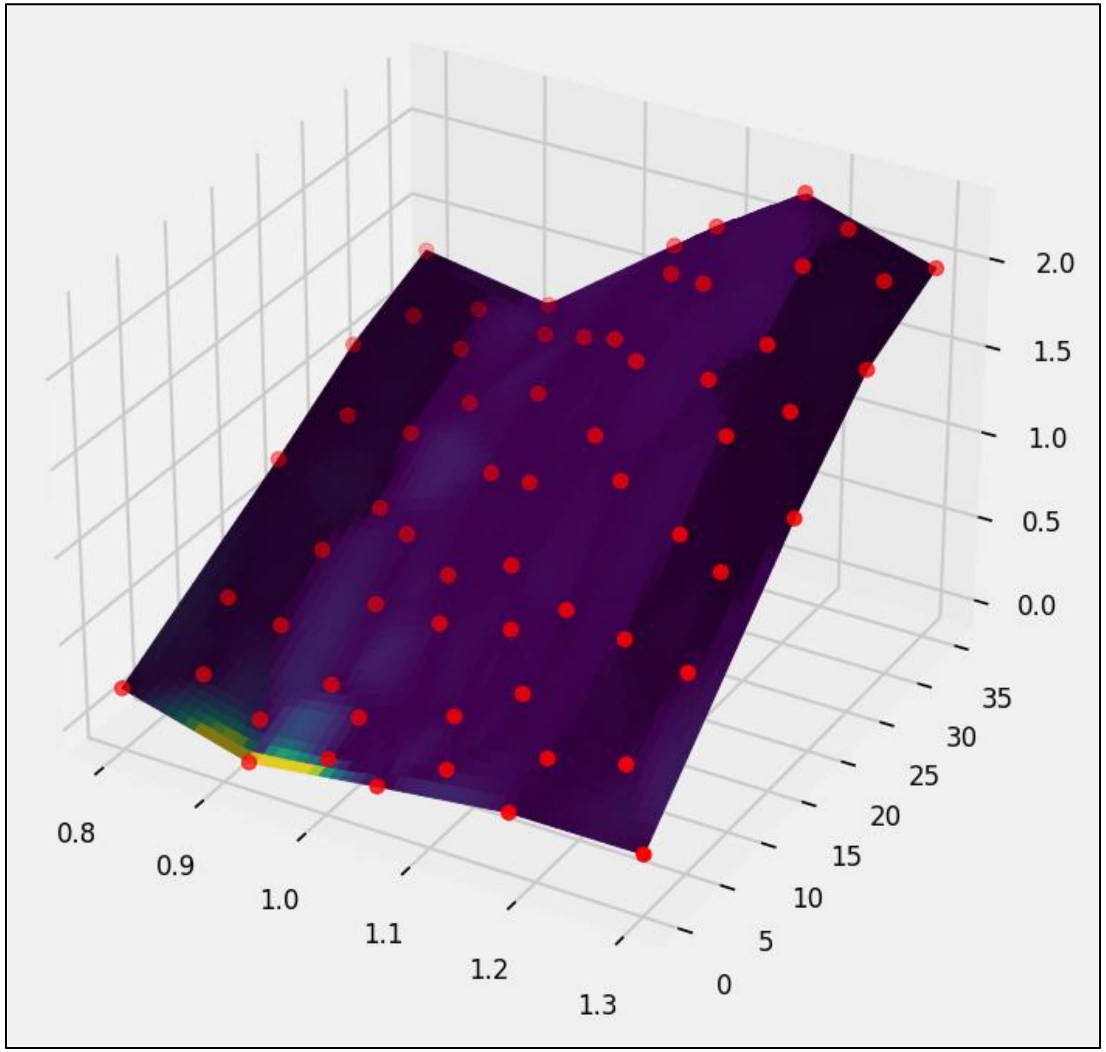



**Figure 9:** Relative error of the volume-filling mesh, demonstrating high error near non-linear boundary

# Conclusion

| Advantages | Limitations |
|---|---|
| Light-weight, iterative algorithm | Currently limited to one dependent output at a time – future work to extend to n-dimensional error sampling strategy |
| Satisfies our purposes for CFD sampling | Points on convex hull cannot be used for error calculations |
| Easy to apply different error metrics | |

Overall, an average of 30% less samples required compared to naive sampling strategies

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# Questions and comments

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